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Lines of Quantised Magnetic Flux and the Relativistic String of the Dual Resonance Model of Hadrons

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Abstract

The quantisation of magnetic flux and the quantisation of electric charge follows from requiring the same invariance properties under time reversal of both classical and quantum systems. The action integral for a line of quantised magnetic flux is the area of the surface traced out in space-time by the motion of the line. It is suggested that the relativistic string of the dual resonance model of hadrons is a line of quantised magnetic flux. Accordingly, quarks have magnetic charge. Assuming quarks of magnetic charge $+g$, $-2g$, baryons are composed of three quarks. States of one, two, four or five quarks will not normally occur. An explanation is given of the failure to produce free quarks.

1. *Introduction*

The quantisation of magnetic flux in an amount $\frac{1}{2}(2\pi\hbar c/e)$ and the quantisation of electric charge in units of e follows from requiring the same invariance properties under time reversal of both classical and quantum systems. e is used here for the smallest possible electric charge, not necessarily the charge on the electron. The action integral for a line of quantised magnetic flux is the area of the surface traced out in space-time by the motion of the line.

It is suggested that the relativistic string of the dual resonance model is a line of quantised magnetic flux. Quarks are identified with ends of magnetic flux lines, and so have magnetic charge. This identification provides an explanation of the failure to produce free quarks.

Finally a comparison is made between this work and other ideas concerning possible magnetic structure of particles.

A preliminary note on this work has been given (Tassie, 1973a).

2. *Quantisation of Magnetic Flux*

Aharonov & Bohm (1959) pointed out that electromagnetic fields have 'non-local' effects in quantum mechanics that do not occur in classical mechanics. If magnetic flux could only exist in quanta of $2\pi\hbar c/e$, the Aharonov–Bohm effect would not occur. Such a possibility was discussed by Aharonov & Bohm (1961), who pointed out that such quantisation of flux is not compatible with the quantum theory of the electromagnetic field as it now stands and that the experiment of Chambers (1960) provided some evidence against flux quantisation. However, the experiment of Chambers and similar subsequent experiments (reviewed by Erlichson, 1970, and Woodilla & Schwarz, 1971) on the observation of the Aharonov–Bohm effect do not rule out flux quantisation due to the presence of a leakage magnetic field.

Conclusive evidence against quanta of $2\pi\hbar c/e$ of magnetic flux was provided by the measurements by Deaver & Fairbank (1961) and Doll & Nābauer (1961) of fluxes $N\frac{1}{2}(2\pi\hbar c/e)$, where N is an integer. These fluxes will be referred to as half-integral fluxes, and fluxes $N(2\pi\hbar c/e)$ will be referred to as integral fluxes. The observations of half-integral fluxes were performed using superconductors. It has seemed that the existence of half-integral magnetic fluxes is intrinsically associated with superconductivity. However, the possibility cannot be discounted that flux quantisation in half-integral fluxes is a universal phenomenon, and that superconductors present only a simple way of observing flux quantisation. Nambu & Jona-Lasinio (1961) have pointed out some similarity between the ground state of a superconductor and the vacuum state in quantum field theory.

The conclusion of Aharonov & Bohm (1961) that flux quantisation is incompatible with the present quantum theory of the electromagnetic field still holds for half-integral fluxes. Therefore, if the quantisation of magnetic flux is a universal phenomenon, considerable changes must be made to electromagnetic theory.

The application of a simple postulate to the Aharonov–Bohm effect leads to the conclusion that magnetic flux occurs only in amounts $N\frac{1}{2}(2\pi\hbar c/e)$ and that electric charge is quantised. The postulate, which is frequently tacitly assumed, is:

Symmetry Postulate. Any quantum system has, at least, all the symmetry properties of the corresponding classical system.

This postulate is applied to a system consisting of a single particle of charge e moving around an impenetrable cylinder which contains magnetic flux. Classically no magnetic force acts on the particle, and the classical system is invariant under the time reversal

$$\begin{aligned} t &\to -t \\ \mathbf{A} &\to \mathbf{A} \end{aligned} \tag{2.1}$$

where $\mathbf{A}$ is the electromagnetic vector potential. Note that this time reversal differs from the symmetry operation of combined time reversal and flux reversal

$$\begin{aligned} t &\to -t \\ \mathbf{A} &\to -\mathbf{A} \end{aligned} \tag{2.2}$$

under which both the classical system and quantum system under consideration are invariant. The quantum system is invariant under time reversal (2.1) only if the magnetic flux inside the cylinder is $N\frac{1}{2}(2\pi\hbar c/e)$ where N is an integer. The argument follows that given by Peshkin, Talmi & Tassie (1961).

Under time reversal (2.1)

$$\mathbf{r} \times M\mathbf{v} \to -\mathbf{r} \times M\mathbf{v} \tag{2.3}$$

It is convenient to choose a cylindrically symmetric gauge, for instance

$$A = \frac{F}{2\pi} \operatorname{grad} \phi \tag{2.4}$$

outside the cylinder, where ϕ is the aximuthal angle around the z-axis which is taken along the axis of the cylinder and F is the magnetic flux which threads the cylinder. The z component of the canonical angular momentum

$$L_z = [\mathbf{r} \times (M\mathbf{v} - e\mathbf{A}/c)]_z \tag{2.5}$$

has eigenvalues $m\hbar$ where m is an integer. But

$$[\mathbf{r} \times M\mathbf{v}]_z = L_z + eF/2\pi$$

and so has eigenvalues $\hbar(m + eF/2\pi\hbar c)$. Equation (2.3) requires that if $\hbar(m + eF/2\pi\hbar c)$ is an eigenvalue of $[\mathbf{r} \times M\mathbf{v}]_z$, then $-\hbar(m + eF/2\pi\hbar c)$ is also an eigenvalue of $[\mathbf{r} \times M\mathbf{v}]_z$, and this requires

$$eF/2\pi\hbar c = N/2 \tag{2.6}$$

where N is an integer. Thus only for fluxes quantised in units of $\frac{1}{2}(2\pi\hbar c/e)$ will the system have the same symmetry properties as both a classical and quantum system. Considering particles with other values of the electric charge shows that for the quantum system to have all the symmetry properties of the corresponding classical system, all charges must be an integer multiple of some fundamental charge, e, and all magnetic fluxes must be an integer multiple of a fundamental flux $\frac{1}{2}(2\pi\hbar c/e)$.

If an arbitrary gauge is used for $\mathbf{A}$, a similar but more complicated argument can be given using gauge-independent symmetry theory (Tassie & Buchdahl, 1964; Buchdahl & Tassie, 1965) with the same conclusion. The argument given by Peshkin & Tobocman (1962) shows that a system of interacting charges, where the charge of the kth particle is $n_k e$, surrounding an impenetrable cylinder containing flux $N\frac{1}{2}(2\pi\hbar c/e)$ is invariant under time reversal (2.1).

The connection between quantised magnetic flux and quantised electric charge is similar to the result of Dirac (1931, 1948) except that Dirac has quantised charges and quantised magnetic monopoles. To satisfy the symmetry postulate, electric charge must be quantised regardless of the existence or non-existence of magnetic monopoles.

3. *Theory of Quantised Flux Lines*

Accepting the symmetry postulate of the previous section, it is necessary to have an electromagnetic theory of quantised fluxes and charges. Some model to provide the flux quantisation can be sought, such as in the analogy of the vacuum state to the superconducting state; so that lines of quantised flux can be envisaged as sheathed in currents arising from virtual pair production in the vacuum. Alternatively, the quantised flux line can be regarded as a tube of very small diameter which is a universe in itself described by general relativity. Such cylindrical universes containing a magnetic field have been considered by Melvin (1964, 1965) and Melvin & Wallingford (1966), and a universe containing a flux $\frac{1}{2}(2\pi\hbar c/e)$ has a radius $\approx 10^{-32}$ cm if e is taken as the charge of the electron.

It seems better for the time being to assume that the thickness of a tube of magnetic flux is negligible, and so consider a line of quantised flux. As a particle traces a world line in space-time, so does a flux line trace out a surface in space-time. This surface will be called the word surface of the line.

In ordinary electrodynamics, the equations of motion for the free electromagnetic field are given by minimising the action integral (Landau & Lifshitz, 1951)

$$S = a \iint F_{ik}^2 \, dV \, dt \tag{3.1}$$

$$F_{ik}^2 = 2(H^2 - E^2) \tag{3.2}$$

For a line of magnetic flux at rest, the action integral is proportional to the area of the world surface of the line. Since the action integral is a Lorentz scalar, for a uniformly moving flux line, the action integral is also proportional to the area of the world surface of the line. By considering the world surface of an arbitrarily moving flux line as made up of infinitesimal tangent surfaces, we conclude that the action integral for a flux line is proportional to the area traced out by the line in space-time. The equations of motion are obtained by the requirement that the area of the world surface be stationary.

The action for a line of quantised flux is

$$S = \beta \int dA \tag{3.3}$$

where dA is the infinitesimal element of area.

4. *Lagrangian Formalism*

Equation (3.3) can be written (Eisenhart, 1949)

$$S = \beta \int \sqrt{g}\, du^1\, du^2 \tag{4.1}$$

where u^1, u^2 are coordinates on the two-dimensional surface.

$$g = \text{determinant}\, g_{ij} \tag{4.2}$$

where the line element on the surface is given by

$$ds^2 = g_{ij}\, du^i\, du^j \qquad i, j = 1, 2 \tag{4.3}$$

In four-dimensional space-time,

$$ds^2 = a_{\alpha\beta}\, dy^\alpha\, dy^\beta \qquad \alpha, \beta = 1 \text{ to } 4 \tag{4.4}$$

$$g_{ij} = a_{\alpha\beta} \frac{\partial y^\alpha}{\partial u^i} \frac{\partial y^\beta}{\partial u^j} \tag{4.5}$$

$$= \frac{\partial y^\alpha}{\partial u^i} \frac{\partial y_\alpha}{\partial u^j} \qquad \text{for } a_{\alpha\beta} \text{ constant} \tag{4.6}$$

Choosing

$$\begin{aligned} y^n &= x, y, z \qquad \text{for } n = 1 \text{ to } 3 \\ y^4 &= ict \end{aligned} \tag{4.7}$$

then

$$a_{\alpha\beta} = \delta_{\alpha\beta} \tag{4.8}$$

We choose

$$u^2 = \tau = ict \tag{4.9}$$

and write

$$u^1 = l \tag{4.10}$$

l is a measure of the length along the flux line.

$$\begin{aligned} g_{11} &= (\partial x_n/\partial l)^2 \qquad n = 1 \text{ to } 3 \\ g_{12} &= g_{21} = (\partial x_n/\partial l)(\partial x_n/\partial \tau) \\ g_{22} &= (\partial x_n/\partial \tau)^2 + 1 \end{aligned} \tag{4.11}$$

$$\begin{aligned} g &= g_{11} g_{22} - g_{12}^2 \\ &= (\partial x_n/\partial l)^2 [1 + (\partial x_m/\partial \tau)^2] - \left(\frac{\partial x_n}{\partial l} \frac{\partial x_n}{\partial \tau}\right)^2 \end{aligned} \tag{4.12}$$

The Lagrangian and Lagrangian density are defined by

$$S = \int L\, dt = \int \mathscr{L}\, dl\, dt \tag{4.13}$$

Then

$$\mathscr{L} = ic\beta g^{1/2} = c\alpha g^{1/2} \tag{4.14}$$

It should be noted that $\mathscr{L}$ is a line density. $\alpha = i\beta$ is real. The Euler–Lagrangian equations

$$\frac{\partial \mathscr{L}}{\partial x_n} - \frac{\partial}{\partial l}\frac{\partial \mathscr{L}}{\partial\left(\frac{\partial x_n}{\partial l}\right)} - \frac{\partial}{\partial t}\frac{\partial \mathscr{L}}{\partial\left(\frac{\partial x_n}{\partial t}\right)} = 0 \tag{4.15}$$

yield

$$\frac{\partial}{\partial \tau}\left\{g^{-1/2}\left[\frac{\partial x_n}{\partial \tau}\left(\frac{\partial x_m}{\partial l}\right)^2 - \frac{\partial x_n}{\partial l}\frac{\partial x_m}{\partial l}\frac{\partial x_m}{\partial \tau}\right]\right\} + \frac{\partial}{\partial l}\left\{g^{-1/2}\left[\frac{\partial x_n}{\partial l}\left(1 + \left(\frac{\partial x_m}{\partial \tau}\right)^2\right) - \frac{\partial x_n}{\partial \tau}\frac{\partial x_m}{\partial l}\frac{\partial x_m}{\partial \tau}\right]\right\} = 0 \tag{4.16}$$

5. *Hamiltonian Formalism*

The momentum canonically conjugate to $x_n(l, t)$ is

$$p_n = \frac{\partial \mathscr{L}}{\partial\left(\frac{\partial x_n}{\partial t}\right)} = \beta g^{-1/2}\left[\frac{\partial x_n}{\partial \tau}\left(\frac{\partial x_m}{\partial l}\right)^2 - \frac{\partial x_n}{\partial l}\frac{\partial x_m}{\partial l}\frac{\partial x_m}{\partial \tau}\right] \tag{5.1}$$

The Hamiltonian line density is given by

$$\mathscr{H} = \sum_n p_n \frac{\partial x_n}{\partial t} - \mathscr{L} = -ic\beta g^{-1/2}\left(\frac{\partial x_n}{\partial l}\right)^2 = c\{p_n{}^2 + \alpha^2(\partial x_n/\partial l)^2\}^{1/2} \tag{5.2}$$

Hamilton's equations

$$\dot{x}_n = \frac{\partial \mathscr{H}}{\partial p_n}; \qquad \dot{p}_n = -\frac{\partial \mathscr{H}}{\partial x_n} + \frac{\partial}{\partial l}\frac{\partial \mathscr{H}}{\partial\left(\frac{\partial x_n}{\partial l}\right)} \tag{5.3}$$

yield

$$\frac{\partial}{\partial t}\{\dot{x}_n[(\partial x_m/\partial l)^2]^{1/2}[1 - c^{-2}(\partial x_m/\partial t)^2]^{-1/2}\} = \frac{\partial}{\partial l}\{c^2(\partial x_n/\partial l)\,[1 - c^{-2}(\partial x_m/\partial t)^2]^{1/2}[(\partial x_m/\partial l)^2]^{-1/2}\} \tag{5.4}$$

However, Hamilton's equations (4.12) are only equivalent to Lagrange's equations (4.8) if the coordinate l is orthogonal to τ,

$$g_{12} = g_{21} = 0 \tag{5.5}$$

6. *Isometric Coordinates*

When the coordinates u^1, u^2 on the surface are chosen so that

$$g_{12} = g_{21} = 0 \tag{6.1}$$

$$g_{11} = g_{22} = \sqrt{g} \tag{6.2}$$

the u^1, u^2 are called isometric coordinates (Eisenhart, 1947, Chapter 3). The coordinate curves u^1 = constant, u^2 = constant divide the surface into small squares to a first approximation. Such a net is called an isometric orthogonal net.

Any family of parallel hyperplanes intersect a stationary surface in curves which together with their orthogonal trajectories form an isometric net (Eisenhart, 1947, Chapter 4). Consider the family of parallel hyperplanes $\tau = ict$ = constant. Then coordinates l can be found so that l, τ are isometric coordinates on a world surface of a flux line, since the world surface is a stationary surface.

For a stationary surface the x^m are solutions of

$$\frac{\partial^2 x_m}{\partial u^{1^2}} + \frac{\partial^2 x_m}{\partial u^{2^2}} = 0 \tag{6.3}$$

where u^1 and u^2 are isometric coordinates, i.e.

$$\frac{\partial^2 x_m}{\partial l^2} - \frac{1}{c^2}\frac{\partial^2 x_m}{\partial t^2} = 0 \tag{6.4}$$

The coordinates of a point on the world surface obey the wave equation. For a flux line at rest, l is the distance along the flux line and a disturbance propagates along the line with the speed of light. For a moving flux line l is not the distance along the line, but

$$ds = [1 - c^{-2}(\partial x_m/\partial t)^2]^{1/2}\, dl \tag{6.5}$$

and a disturbance propagates along the line with a speed less than that of light.

7. *Commutation Relations*

We now consider the replacement of the usual description of electrodynamics by a description in terms of quantised flux lines and show that this leads to the need for the introduction of commutation relations.

Electric fields can be considered to arise from the motion of magnetic flux lines. A net electric field but zero magnetic field can be obtained by considering two sets of magnetic flux lines in opposite directions perpendicular to the electric field with one set of flux lines moving. This situation is analogous to the description of currents as moving charges. A net current with zero charge density is obtained by charges of one sign moving past charges of the other sign. The analogy is a close one, for we have quantised charge and quantised magnetic flux which in motion can give rise to arbitrary current and arbitrary electric flux respectively.

The description of the interaction of a quantised magnetic flux line with a charged particle is complicated, and is entirely through the non-local Aharonov–Bohm effect. A 'uniform' magnetic field will be described by a uniform distribution of flux lines. Then it can be shown that the theory of motion of quantum mechanical particles through fixed flux lines is in contradiction with experiment. For the motion of the charged particle around each single flux line is invariant under time reversal, and remains invariant under time reversal for many flux lines. Then the motion of a charged particle through a 'uniform' magnetic field is invariant under time reversal. Experimentally this is not so; an electron moving through a uniform magnetic field will not retrace its path if its velocity is reversed. Thus the theory does not agree with experiment. However, the theory is also inconsistent, as $\hbar$ occurs in the relation between charge and flux, but we have a classical description of the motion of the flux line, which corresponds to taking $\hbar = 0$ in considering the motion of the flux line.

Working in the Schrodinger picture, we postulate the following commutation relations, in analogy to the usual canonical commutation relations of quantum field theory

$$[p_m(l), p_n(l')] = 0 \tag{7.1}$$

$$[x_m(l), x_n(l')] = 0 \tag{7.2}$$

$$[p_m(l), x_n(l')] = \frac{\hbar}{i}\delta_{mn}\,\delta(l - l') \tag{7.3}$$

8. *Magnetic Monopoles*

In usual electromagnetic field theory

$$\operatorname{div}\mathbf{B} = 0$$

and magnetic monopoles do not occur. The corresponding line theory is that of endless flux lines.

If flux lines have ends, the ends are magnetic monopoles. The end of a flux line of unit flux $\frac{1}{2}(2\pi\hbar c/e)$ has a pole strength or magnetic charge

$$g = \tfrac{1}{4}\hbar c/e$$

which is one-half of that of the Dirac magnetic monopole (Dirac, 1931, 1948; Amaldi & Cabibbo, 1972). This magnetic monopole is very different from that of Dirac. The Dirac monopole is attached to a string which is a singularity in the vector potential which is physically unobservable and otherwise is a source of flux

$$\int \mathbf{B}.d\mathbf{S} = 2\pi\hbar c/e$$

All the magnetic field of the monopole proposed here is contained in the quantised flux line attached to it. A method of detection of these monopoles using a superconducting coil has been proposed previously (Tassie, 1965). However, if the considerations of Section 9 are correct, free magnetic monopoles would not normally occur.

Experiments have established that magnetic monopoles, of either the Dirac type or the type proposed here, are extremely rare (Amaldi & Cabibbo, 1972).

9. *Hadrons and Quarks*

Nambu (1970) and Susskind (1970) have obtained amplitudes for the interactions of hadrons similar to the Veneziano amplitude (Veneziano, 1968) using a dual resonance model in which the resonances are identified with excitations of a relativistic string of finite length. In the case of mesons, a quark is embedded in one end of the string and an antiquark at the other end. Goto (1971), Minami (1972), and Goddard *et al.* (1972) have shown that the equations of motion of the relativistic string of the dual resonance model can be obtained from an action integral proportional to the area swept out by the string in space-time. The action integral is identical with that of equation (4.1) for the line of quantised magnetic flux. Goto (1971) and Minami (1972) show that the subsidiary conditions imposed by Virasoro (1970) in order to eliminate unphysical states in the dual resonance model correspond to the conditions of equations (6.1) and (6.2).

The close mathematical similarity of the relativistic string of the dual resonance model with the theory of Sections 3–7 suggests that the relativistic string should be identified with a line of quantised magnetic flux. A meson is identified with a finite length of string. Because of the direction of the magnetic flux, the two ends of the string are not equivalent and can be identified with quark and antiquark (see Fig. 1a). Thus the quark is also a magnetic monopole of the type discussed in Section 8.

The baryon consists of a bound state of three quarks as shown in Fig. 1c if it is assumed that there are two types of quarks with magnetic charge $+g$, $-2g$. The quark of magnetic charge $-2g$ consists of a change of direction of magnetic flux in a line.

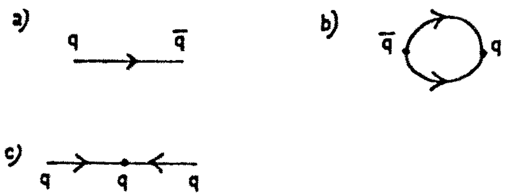

Figure 1.—q and $\bar{q}$ denote quark and antiquark respectively. (a) and (b) mesons, (c) baryon. The arrowheads show the direction of magnetic flux in the quantised flux lines.

This picture of the structure of hadrons provides an explanation for the apparent contradiction between the success of the naive quark model which indicates that the quark is only weakly bound within the hadron (Dalitz, 1968, 1970; Kokkedee, 1969; Morpurgo, 1970; Flamm, 1970) and the failure to produce quarks experimentally (Morpurgo, 1970; Giacomelli, 1972). As a quark is extracted from a hadron, it trails a string of quantised flux. Breaking the string is equivalent to creating a meson, and so the lowest energy required to break the string is just the rest energy of the pion. So that an attempt to produce free quarks in a high-energy collision just leads to the break-up of the string into pieces, corresponding to the production of mesons. Thus, with this picture, it is very difficult, if not impossible, to produce a free quark because it would be the end of an infinitely long string which is easily broken. Similarly states of two, four, five, seven, etc., quarks would have infinite magnetic strings attached, and so would not normally be expected to occur.

The success of the parton model (Feynman, 1972; an elementary account is given by Tassie, 1973b) suggests that in collisions of hadrons, quarks recoil as if weakly bound, although no free quarks are produced. Such behaviour is expected in this picture, as the string attached to the recoiling quark breaks.

Mesons can also be made of a quark of magnetic charge $-2g$ and its antiquark corresponding to the loop structure of Fig. 1b. Breaking the two flux lines in such a meson yields a baryon–antibaryon pair. It is expected that the physically occurring mesons are quantum-mechanical superpositions of the structures shown in Fig. 1a and 1b.

It is assumed here, that for each value of the magnetic charge, g, $-2g$, there is the usual $SU(3)$ triplet of quarks (Gell-Mann, 1964).

10. *Discussion*

The quantised flux lines proposed here have some similarity with those proposed by Jehle (1970, 1972). For instance, Jehle points out that the moving flux line gives rise to an electric field, and considers the electromagnetic fields arising from particular configurations of flux lines. However, Jehle only considers flux lines forming closed loops. Also his motivation for introducing quantised flux lines has no connection with invariance under time reversal, and his flux lines contain flux $2\pi\hbar c/e$, twice the flux of the flux lines considered here.

Various suggestions have been put forward (Schwinger, 1969; Han & Biedenharn, 1970; Barut, 1971, 1972) that hadrons are built up of magnetic monopoles in such a way that the total magnetic charge of a hadron is zero, using magnetic monopoles similar to those proposed by Dirac (1931, 1948). The magnetic monopoles suggested here are very different from those of Dirac, as pointed out in Section 8.

Whether any of these imaginative pictures of hadronic structure in terms of magnetic effects have any ultimate validity remains to be seen. The scheme suggested here has the advantage of fitting in with the dual resonance model and the parton model of hadrons.

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